

# MEI Structured Mathematics

## Practice Comprehension Task - 2

### Do trains run late?

There is a popular myth that trains always run late. Actually this is far from the case. All train companies want their trains to be on time. A single late running train may well cause delays to other trains, for example those running just behind it. This creates customer dissatisfaction and can result in compensation having to be paid.

Companies are required to submit weekly reports on the punctuality and reliability of their trains; an extract from such a report, for one particular line, is given below in Table 1. The numbers in the column headed “run” are the numbers of trains that ran each day. The figures in the six right hand columns give the numbers and percentages of trains arriving at their final destination by certain times; thus on Sunday 22/06/97, 22 trains arrived within 10 minutes of the scheduled time and this was 73.3% of the 30 trains that ran that day. 5  
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Day	Date	Run	Early	Right time	Time to 5	Time to 10	Time to 30	Time to 60
Sunday	22/06/97	30	4 13.3%	6 20.0%	17 56.7%	22 73.3%	28 93.3%	30 100%
Monday	23/06/97	41	22 53.7%	24 58.5%	30 73.2%	35 85.4%	41 100%	41 100%
Tuesday	24/06/97	42	21 50.0%	23 54.8%	30 71.4%	34 81.0%	40 95.2%	42 100%
Wednesday	25/06/97	42	25 59.5%	27 64.3%	38 90.5%	40 95.2%	42 100%	42 100%
Thursday	26/06/97	42	17 40.5%	20 47.6%	26 61.9%	37 88.1%	42 100%	42 100%
Friday	27/06/97	42	23 54.8%	27 64.3%	34 81.0%	38 90.5%	41 97.6%	42 100%
Saturday	28/06/97	31	14 45.2%	14 45.2%	20 64.5%	23 74.2%	29 93.5%	31 100%

**Table 1. Performance Statistics for the trains on one line for the week ending 28/06/97**

These reports are summarised in the performance tables published on main line stations.

There are various reasons for delays to trains, many of them relating to passenger safety. Under the wrong conditions, for example wet leaves on the rails, the friction between steel wheels and steel rails may fall to a very low value, causing a train to have a much longer stopping distance than usual and so necessitating slower running speeds. There is always routine maintenance in progress and trains are usually required to slow down over the section where work is being one. 15

Clearly it would be quite unrealistic to construct a simple timetable on the basis that no delay will occur. That would ensure that all trains run late.

In practice train companies work with two timetables: 20

- The *Working Timetable* for a train assumes that it travels at the maximum permitted speed over every section of the track (subject to predicted delays like time lost by slowing down for station stops and subsequently picking up speed). The working timetable is not published.

- The *Passenger Timetable* allows for a realistic level of delay by scheduling the train to take some minutes longer than the working timetable. If those delays do not occur the train appears to be early. Most passengers would prefer to be told that their journey will take rather longer and then to arrive on time, or even early, than to be made promises that are seldom kept. The Passenger Timetables for the trains on a route are assembled in the published timetable which you can get at any station. 25 30

The difference in times between the Passenger and Working Timetables is called the *Engineering Allowance*.

Thus the allowed time for a journey may be modelled by the equation

$$T = T_0 + E, \quad 35$$

where  $T_0$  is the shortest possible journey time and  $E$  is the Engineering Allowance.

At first sight it would seem that all a train company needs to do is to increase the Engineering Allowance and a higher proportion of trains will be on time or early. It is not, however, quite as simple as that.

Most trains stop at intermediate stations and a train cannot leave one of those early; otherwise intending passengers would complain about being left behind. Increasing the time between intermediate stations amounts to slowing the train down and thereby providing a less good service. 40

You can get some idea of how the Engineering Allowance is applied in practice by looking inside a published timetable. The figures in Table 2 refer to two scheduled trains (at the time of writing this article), the 0935 from London Paddington to Plymouth and the 0735 going the other way, both operated by Great Western Trains. The trains stop at the same intermediate stations and both a scheduled to take 3 hours 35 minutes. However, the times allowed between any pair of stations are by no means the same. 45

The largest differences are between the times allowed for the final stage of each journey and the corresponding first stage of the journey in the opposite direction. Thus Reading to London Paddington is scheduled to take 33 minutes compared to only 26 minutes from London Paddington to Reading. A similar difference occurs between Totnes and Plymouth. Clearly the practice is to apply much of the Engineering Allowance on the final stage of a journey. 50 55

	<b>To Plymouth</b>	<b>To London</b>
London Paddington	0935	1110*
Reading	1001	1037
Totnes	1238	0802
Plymouth	1310*	0735

**Table 2. Timetables for two Great Western trains between London Paddington and Plymouth.**

- Notes: (i) Times given are departure times except those marked with \*.  
(ii) There are intermediate stations at which both trains stop, but they are not relevant to this article.

Is it possible then to deduce the working timetable for the Plymouth to London Paddington line just by looking at the timetables for these two trains, one in each direction, and taking the lesser time between each pair of stations? 60

On the basis of what we have said so far this would appear to be the case. However, when you look at the times of other trains on the route it becomes clear that there are other considerations to be taken into account. Table 3 gives the times of all Great Western trains stopping at both Newbury and Reading on one day. 65

Newbury - Reading	Reading - Newbury
19 18 20	16 15 16 15
19 19 22	17 16 16 17

**Table 3. Train times between Reading and Newbury.**

Clearly it is possible for a train to run between Reading and Newbury in 15 minutes, but most of them are expected to take a few minutes more than this.

Such variability is not predicted by the simple model we have developed so far. The reason for this variability is that two other allowances are also made.

The *Pathing Allowance*,  $P$ , is time added on to allow for known movements of other trains. It may be necessary to slow a train down because of a slower moving scheduled train in front (for example a freight train); there may be a delay while joining another line or there may be a delay waiting for a platform to become available at a major junction station. 70

The *Route Allowance*,  $R$ , is the extra time, usually quite small, allowed for a particular train to change track. This may, for example, be needed because a station uses only one of its platforms late at night. 75

Thus the model for the allowed time,  $T$ , is now refined to become

$$T = T_0 + E + P + R.$$

Looking at the published timetable allows you to make an estimate for  $T_0$ , the shortest possible journey time between any pair of stations. You can be certain that a train can cover the distance in the shortest journey time in the timetable. However, since you cannot be sure that, even in that case, the values of all the allowances  $E$ ,  $P$  and  $R$  are zero, this may in fact not be the shortest possible time. It does, however, give you an upper bound for  $T_0$ . 80

You can then go on to obtain an upper bound estimate for the shortest possible total journey time by adding together your upper bounds for the values of  $T_0$  between pairs of stations on the way. 85

The trains used to illustrate this article stop at all the major stations on the line. Many trains have fewer stops and so take less time to reach their destination

**Questions.**

- 1 This question refers to the data in Table 1.
- (i) One of the performance targets is that 90% of trains should arrive at their destination no more than 10 minutes after the scheduled time.  
State the days of this week on which this target was met. [1]
  - (ii) What percentage of the trains arrived early? [2]
  - (iii) What percentage of trains which arrived late arrived more than 30 minutes late? [2]
- 2
- (i) In line 52 it is stated that the times of the trains given in Table 2 between Reading and London are 26 minutes and 33 minutes. Justify this assertion. [1]
  - (ii) Give the times that the trains could arrive at their destinations if the Engineering Allowance between Reading and London and Totnes and Plymouth were removed. [3]
  - (iii) The timetable below refers to two trains run over a different route by a different company. Determine whether this company also applies an Engineering Allowance on the final stage of the journey. [3]

	To Sheffield	To London
London	0925	1546*
Luton	0946	1522
Chesterfield	1125	1340
Sheffield	1142*	1327

(Note: As before, times marked with a \* are arrival times and the trains stop at other stations en route.)

- 3 Reading is a major junction on the line, but Newbury is not. What does Table 3 tell you about any possible Pathing Allowance added at Reading? [1]
- 4 One Sunday recently on a journey north to Nottingham from London it was announced that the train would stand in Bedford Station for an extra 10 minutes. After the period of time the train moved on, changing tracks onto the slow line. At this particular point in the route the single slow line is separate from the fast lines. Give a possible explanation for this delay that would not have occurred had it not been for engineering works on the fast tracks. [1]
- 5 The table below is of the route north to Nottingham. Built into the timetable is an Engineering Allowance and a Pathing Allowance.  
Deduce the value of  $T_0$ , the shortest possible time between Nottingham and London. [4]

<b>Nottingham</b>	0852	<b>London</b>	0830
<b>Beeston</b>	0858	<b>Luton</b>	0853
<b>Leicester</b>	0935	<b>Leicester</b>	0956
<b>Luton</b>	1036	<b>Beeston</b>	1019
<b>London</b>	1104	<b>Nottingham</b>	1028

**Answers.**

<b>1</b>	<b>(i)</b>	Wednesday and Friday	B1 <b>1</b>	
	<b>(ii)</b>	Trains arriving early = 126 Number of trains run = 270 Percentage arriving early = $126/270 = 46.7\%$	M1 A1 <b>2</b>	
	<b>(iii)</b>	No. arriving early or right time = 141 No. arriving late = $270 - 141 = 129$ No. arriving more than 30 minutes late = 7 Percentage of those arriving late arriving more than 30-mins late = $7/129 = 5.4\%$	M1 A1 <b>2</b>	
<b>2</b>	<b>(i)</b>	From London dep 0935, arrive 1001 giving time of 26 mins From Reading dep 1037, arrive 1110 giving 33 mins	B1 <b>1</b>	Both
	<b>(ii)</b>	Dep. Reading 1037 with journey time of 26 mins gives arrival time of 1103  Times between Totnes and Plymouth are 32 and 27 mins So Dep Totnes at 1238 with journey time of 27 mins gives arrival time of 1305	B1  M1 A1 <b>3</b>	
	<b>(iii)</b>	Yes. Times for Chesterfield - Sheffield are 13 and 17 mins Times for London - Luton are 21 and 24 mins	M1 A1 A1 <b>3</b>	
<b>3</b>		Times should all be the same. If not then since Reading is the major junction you would expect times for the Newbury - Reading journey to be longer. This is shown by the data in the table.	B1 <b>1</b>	
<b>4</b>		Engineering works meant that the fast lines could not be used. But since there was only one slow line this was possibly used by a southbound train. So this line was not available for 10 mins. Thus a 10 mins pathing allowance was added.	B1 <b>1</b>	
<b>5</b>		Nottingham - Beeston 6 Beeston - Leicester 23 Leicester - Luton 61 Luton - London 23  Gives shortest time = 113 mins (i.e. Train to London dep 0852, arrive 1045 Train to Nottingham dep. 0830, arrive 1023)	M1 A1  M1 A1 <b>4</b>	