LANDAU FORTE COLLEGE DERBY

A consistent approach to maths skills

Contents

Introduction:

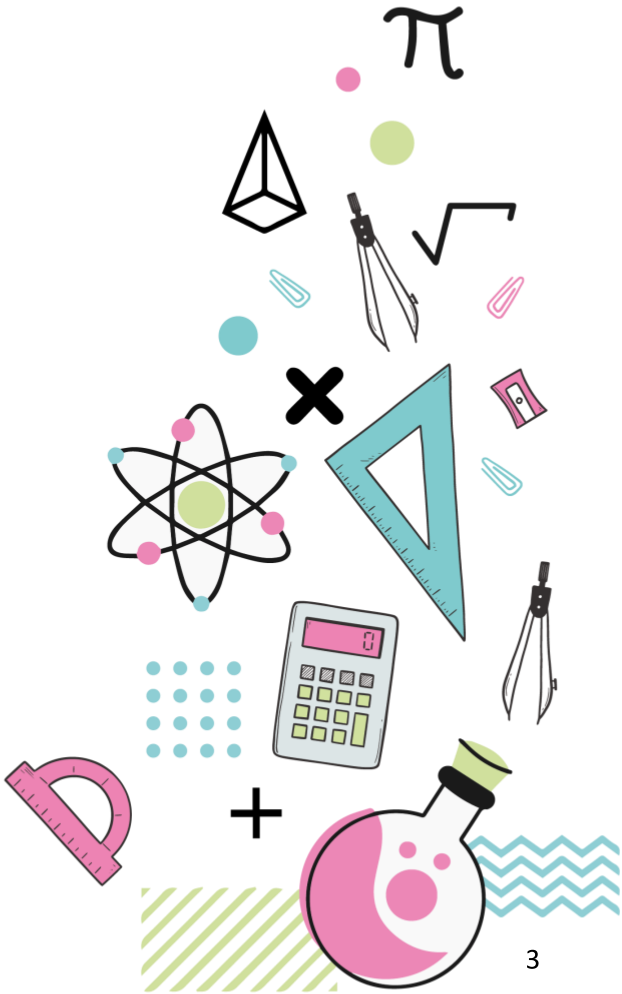
Maths is embedded in all subjects and underpins the whole curriculum.

In KS3 students are required to read/write dates in history and they are required to underline dates and titles in all subjects – meaning they must be able to accurately use a ruler – a key mathematical skill. KS3 students will also be required to convert accurately between units of measure in a number of subjects including Maths, Design & Technology, Art and Science.

As students move into KS4 they will also need an understanding of various statistical measures/charts to display information – in Geography they may need to draw interpret a number of graphs. In Business students will use key formulae and need algebraic rearranging skills and in Computer Science students will use various algorithms and explore a variety of number systems.

With maths clearly being an integral part of all subjects, a consistent approach is essential to support students understanding and avoid confusion.

This booklet will outline some cross-curricular mathematical skills and a consistent approach to them, including: the correct language to use, step-by-step processes and examples.



Numeracy

1.1 Place Value

Place value is the value of each digit that appears in a number. Understanding place value helps you to work out the value of a number.

Correct language:

It is important that the headings for place value are as identified/referred to as in the step-by-step process below. The 'ones' column must no longer be referred to as the 'units' column.
Thousands and Thousandths are different to each other (as shown below) and this needs to be made clear.

Step-by-step process:

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones	•	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths	Millionths
M	Hth	TTh	Th	H	T	O	•	t	h	th	tth	hth	m

Example:

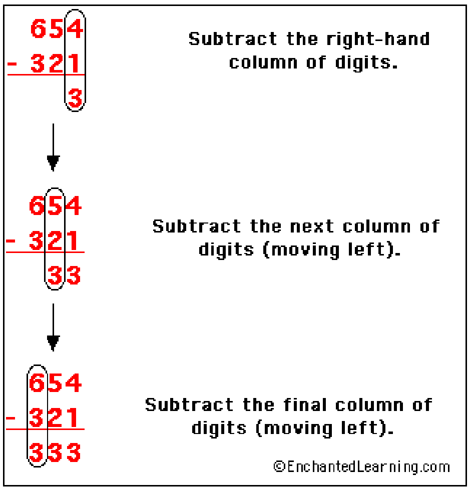
NUMERALS	PLACE VALUE	VALUE
4826	4 Thousand	4000
4826	8 Hundred	800
4826	2 Tens	20
4826	6 Ones	6

1.2Addition and Subtraction

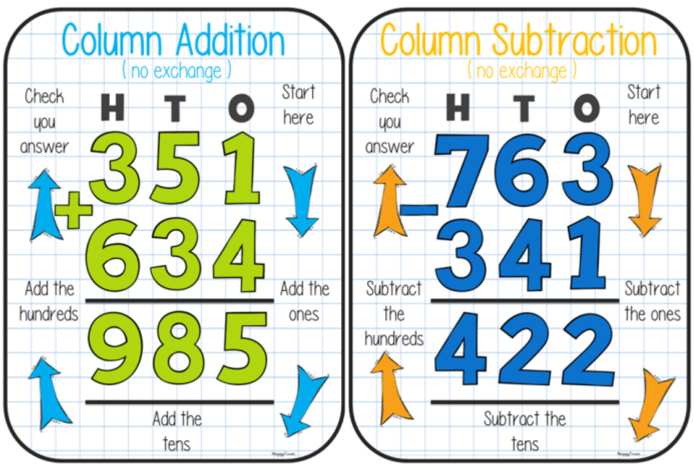
Addition and subtraction are two of the four fundamental maths operations. Being able to add and subtract numbers confidently plays a key role not only in maths but in a number of subject areas.

Correct language:
Addition, Subtraction, Column, Digits

Step-by-step process:
The column methods are to be used for both addition and subtraction for calculations of numbers greater than 1x1 digit.



Example:



1.3Multiplying 2x2 digit or 2x3 digit numbers

Multiplication is one of the four elementary mathematical operations of arithmetic, with the other ones being addition, subtraction, and division.

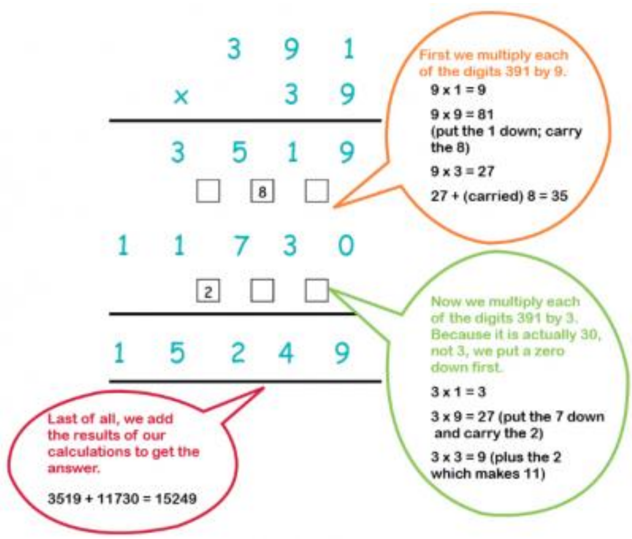
Correct language:

Multiplication, digits, long multiplication.

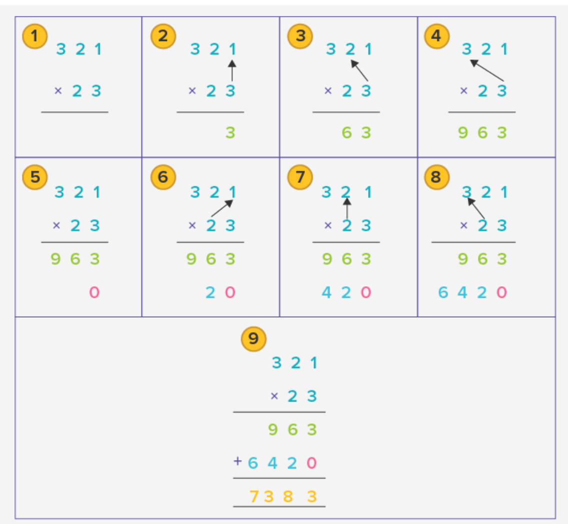
When using the method outlined in the step-by-step process below, when a zero is put down first, refer to this zero as a '*place holder*'.

Step-by-step process:

All students taught to multiply 2x3 digit or 2x3 digit numbers (and beyond) using the standard algorithm (long multiplication).



Example:



<u>Scaffold (alternate approach):</u>

Grid Method:

123 × 5

×	100	20	3
5	500	100	15

```
    500
+   100
+    15
    ---
    615
```

1.4Dividing numbers with 1 or 2 digit divisors

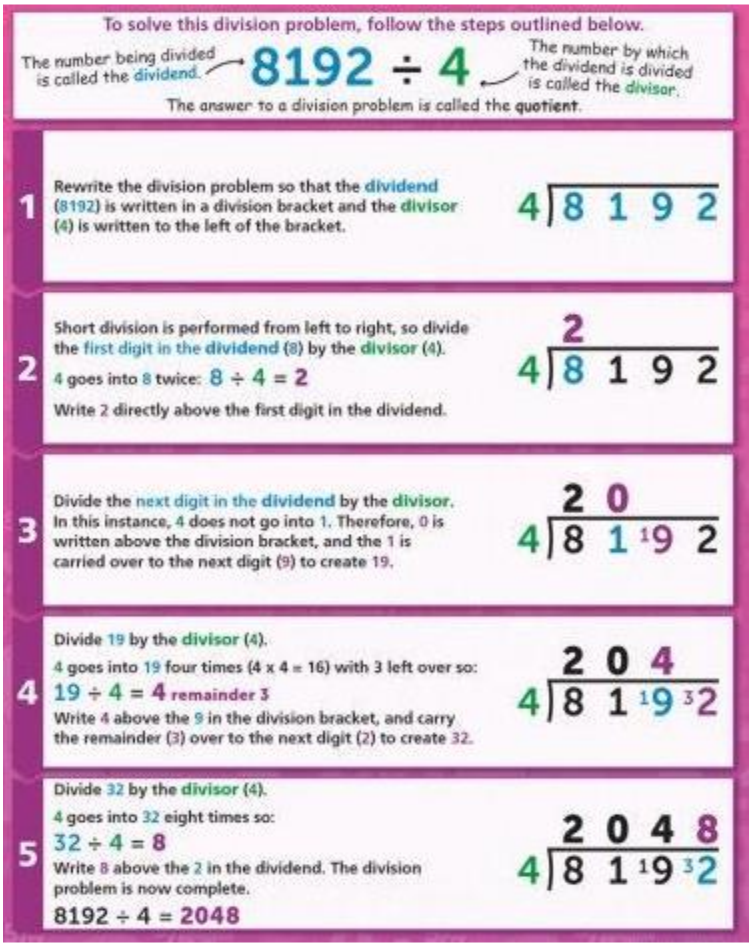
Division in maths is the process of breaking a number up into equal parts, and finding out how many equal parts can be made. For example, dividing 15 by 3 means splitting 15 into 3 equal groups of 5.

Correct language:

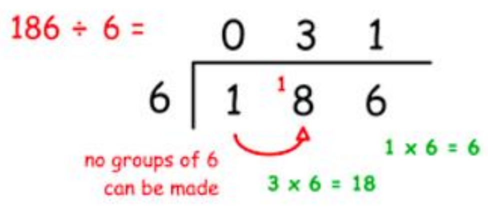
Division, dividend/divisor, 'bus stop'

Step-by-step process:

All students are taught to divide numbers with 1 or 2 digit divisors by using short division – often referred to as 'Bus Stop'.



Example:

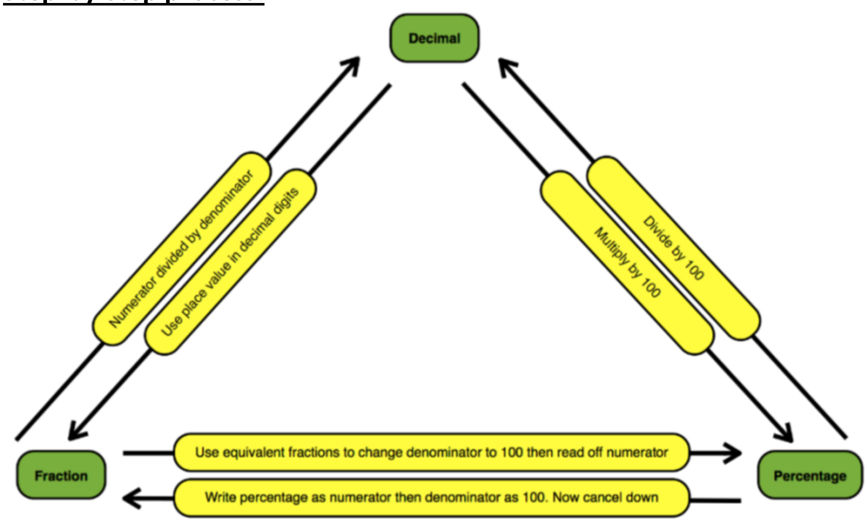


1.51 FDP – Converting between them

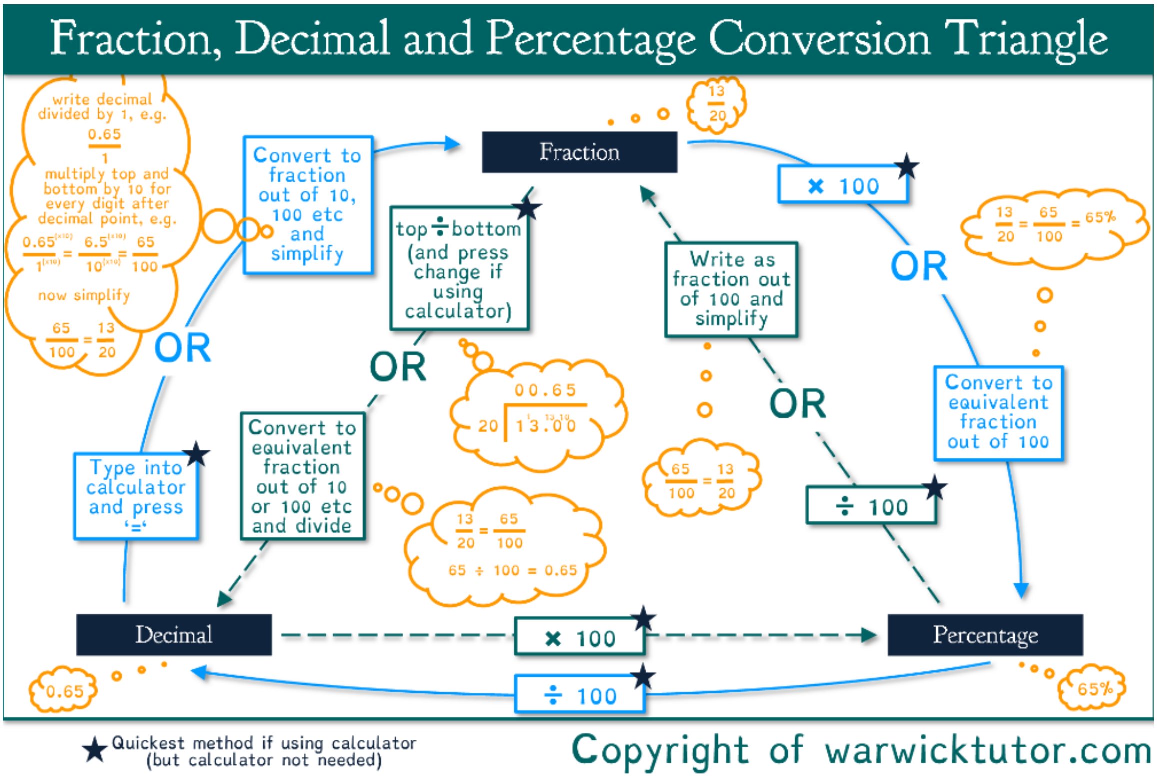
FDP stands for fractions, decimals and percentages. Students will need to convert between fractions, decimals and percentages at various points across KS3 and KS4 in a number of subjects.

Correct language:
Fractions, decimals, percentages, conversions, equivalent.
Refer to the 'top' and 'bottom' of a fraction as the numerator and denominator.

Step-by-step process:



Example:



1.52 FDP – Finding a fraction of a number

Finding a fraction of a number is something that will arise in a number of subjects across the curriculum. It refers to problems such as "find $\frac{2}{3}$ of 12".

Correct language:

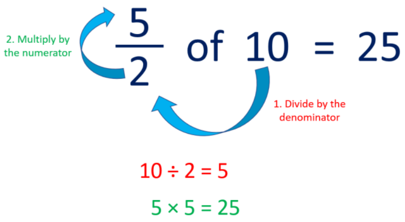
Refer to the 'top' and 'bottom' of a fraction as the numerator and denominator.

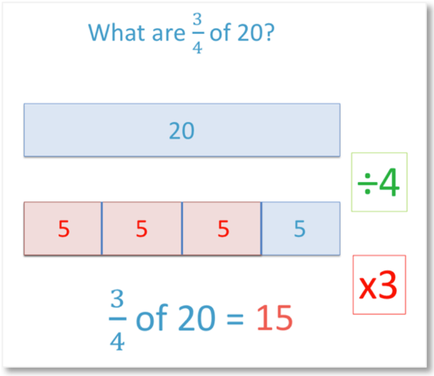
Step-by-step process:

Students will find the fraction of a number the same way across all subjects – for improper and proper fractions. Students will divide by the denominator and then multiply by the numerator. Some students may need the use of a bar model to support with this.

Step 1: Divide the number by the denominator
Step 2: Multiply the number by the numerator

Examples:





1.53 FDP – Finding a number out of another as a percentage

For example, “2 out of 7 people living in a city are living in poverty, what is this as a percentage?”.

Correct language:
Refer to the ‘top’ and ‘bottom’ of a fraction as the numerator and denominator.

Step-by-step process:
Students will use the formula outlined below to find one number out of another as a percentage, this could be to convert a raw test score into a percentage.

$$\frac{Numerator}{Denominator} \times 100$$

Example:
Jay scored 68 out of 80 on his maths test, write this as a percentage.

$$\frac{Numerator}{Denominator} \times 100 = \frac{68}{80} \times 100 = 85\%$$

1.54 FDP – Finding a percentage of a number

A percentage refers to something expressed as an amount out of 100. Finding a percentage of a particular amount can be found a number of ways. A consistent approach to this is outlined in the step-by-step process below.

Correct language:
Use the word multiplier (decimal multiplier) to describe the decimal equivalent of a percentage that the amount is multiplied by.

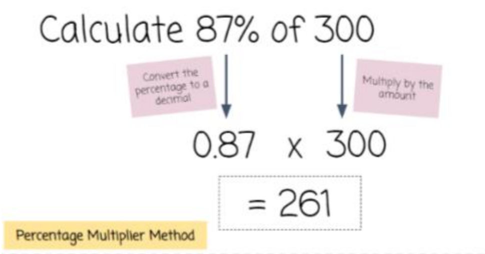
Step-by-step process:

Students will use the multiplier method to find a percentage of an amount.

Step 1: Convert the percentage to a decimal by dividing it by 100 (this is called the multiplier)

Step 2: Multiply the number by the multiplier

Example:



Scaffold (alternate approach):

Some students (or in some scenarios) may need to find the percentage of an amount by finding 10% (by dividing the number by 10) and/or finding 1% (by dividing the number by 100) and building this up to find the required percentage as below:

Find 35% of 60:

10% = 60 ÷ 10 = 6
5% = 6 ÷ 2 = 3

So 35% = 3x10% + 5%
= 3x6 + 3
= 18 + 3
= 21

FDP – Increasing/Decreasing by a percentage

Finding a percentage increase/decrease for a particular amount can be found a number of ways. A consistent approach to this is outlined in the step-by-step process below.

Correct language:
Use the word multiplier (decimal multiplier) to describe the decimal equivalent of a percentage that the amount is multiplied by.

Step-by-step process:
Students will calculate a percentage increase/decrease by using decimal multipliers.

% increase:
Step 1: 100% + percentage
Step 2: Divide this new percentage by 100 to get the decimal multiplier
Step 3: Multiply your number by the decimal multiplier

% decrease:
Step 1: 100% - percentage
Step 2: Divide this new percentage by 100 to get the decimal multiplier
Step 3: Multiply your number by the decimal multiplier

Example:

Using Multipliers for Percentage Changes

If an amount is increased by x% the new amount is $(100 + x)$% of the original amount

Increase 9000 by 15%

100% + 15% = 115%

115% = 1.15

9000 x 1.15 = 10350

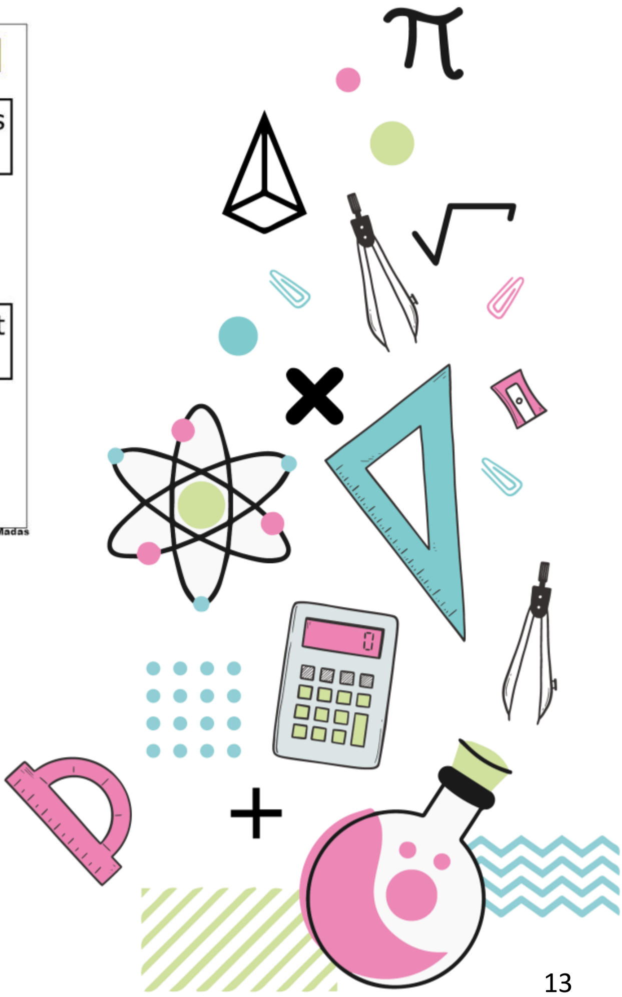
If an amount is decreased by x% the new amount is $(100 - x)$% of the original amount

Decrease 4500 by 12%

100% – 12% = 88%

88% = 0.88

4500 x 0.88 = 7920



1.55 FDP – Finding a percentage change

This refers to calculating the percentage in which a value has been increased or decreased by. There are a number of ways in which this may be approached but a consistent method is outlined below.

Correct language:
Percentage change, increase/decrease.

Step-by-step process:
Students will calculate a percentage change (increase or decrease) by using the method below.

$$Percentage\ Change = \frac{change}{original\ amount} \times 100$$

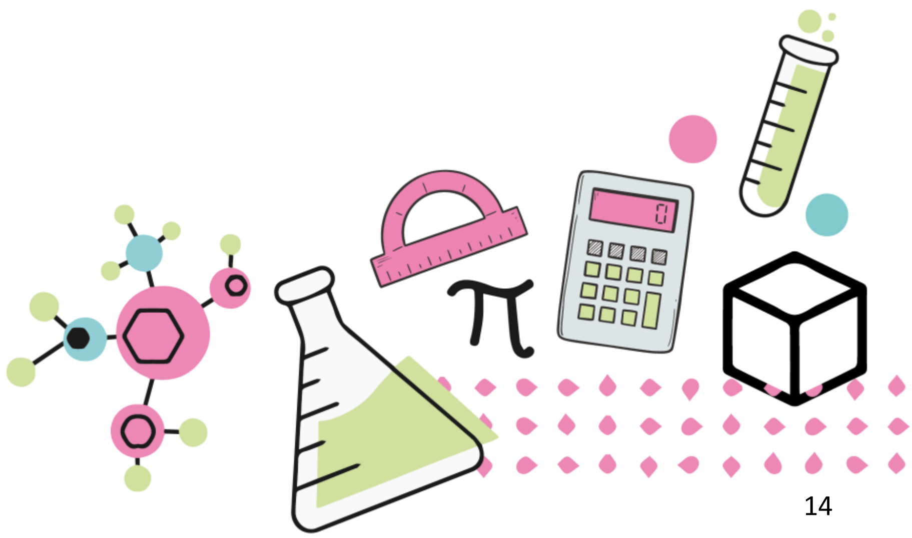
Example:

A piece of jewellery has increased in value from £1560 to £2760. Calculate the percentage change (increase).
Change = £2760 - £1560 = £1200

Percentage change =

$$= \frac{£1,200}{£1,560} \times 100\%$$

Percentage change =76.9%



1.56 FDP - Compound interest

Compound interest occurs when previously earned interest is added to the principal amount invested or borrowed. It is commonly described as "interest earned on interest".

Correct language:

Use the word multiplier (decimal multiplier) to describe the decimal equivalent of a percentage that the amount is multiplied by.

Step-by-step process:

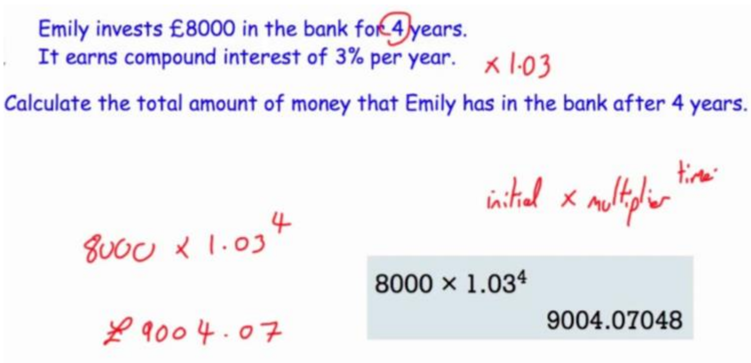
Students will use percentage multipliers in compound interest questions as shown below.

$$New\ value = Initial\ value\ \times multiplier^n$$

Where n is the number of times the interest is applied eg. number of years
The multiplier is calculated as follows:

Step 1: 100% + interest rate
Step 2: Divide this new percentage by 100 to get the decimal multiplier

Example:



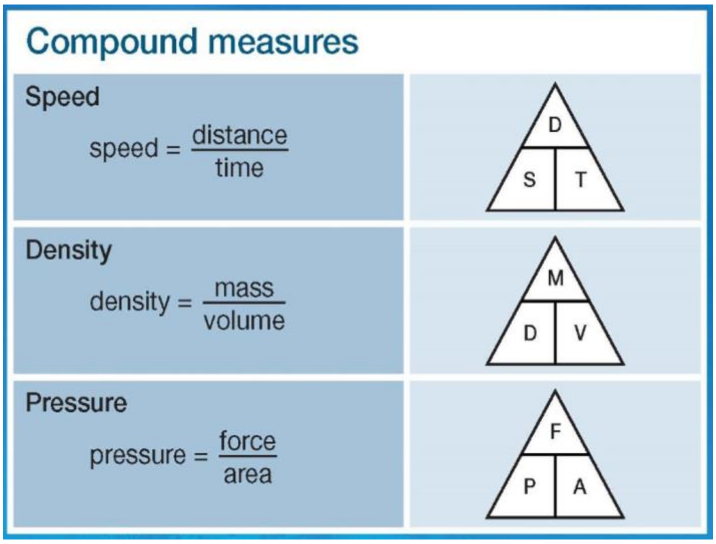
Algebra

2.1 Using Compound measures

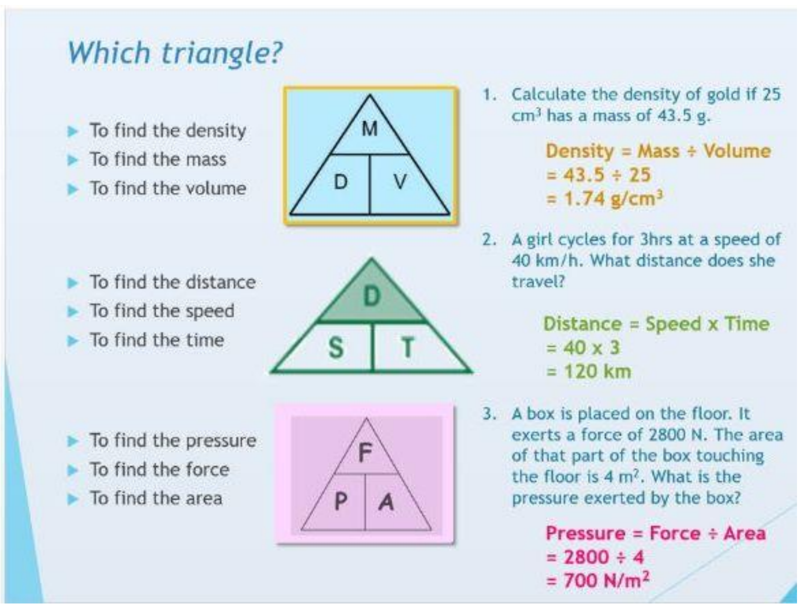
A Compound measurement is a measurement that uses more than one quantity. Examples include density measurements, speed measurements and rates of pay. Density is calculated by mass ÷ volume. So density is therefore written as mass per volume.

Correct language:
Density, speed, pressure, mass, volume, distance, time, force, area, compound.

Step-by-step process:
Use the formula triangles to help remember the compound measures: Speed, Distance, Time; Density, Mass, Volume and Force, Pressure, Area.



Example:



2.2 Rearranging formulae

Typically we rearrange equations and formulas by using inverse operations to make one variable the subject of the formula. The subject of the formula is the single variable that is equal to everything else. i.e. the term by itself on one side of the equal sign.

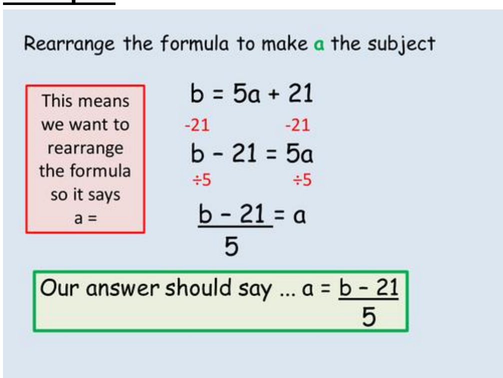
Correct language:
Inverse operations, Balanced, Variable, Formulae, Equations.

Step-by-step process:
Inverse operations should be used when rearranging formulae to make another variable the subject. If there is a question where the values of some variables have been given and a variable that is not the subject is to be found, then substituting and using inverse operations to find the value of the variable may be the most appropriate method.

The formula must stay 'balanced' so an operation acted on one side of the formula must also be acted on the other side.
Inverse operations:

Operation	Inverse
+	−
−	+
×	÷
÷	×
x^2	$\sqrt{x}$

Example:



OR $E = \frac{1}{2}mv^2, find\ v\ when\ E = 250\ and\ m = 2$

Substitute in number first : $250 = \frac{1}{2} \times 2 \times v^2$

$$250 = v^2$$

$$\sqrt{250} = v$$

$$\underline{15.8\ (1dp) = v}$$

2.3 Gradients

On a graph, the gradient is a measure of the steepness of a line (or a point on a curve), and is calculated by dividing the vertical change by the corresponding horizontal change. It represents the rate at which the variable plotted on the vertical axis changes with the variable plotted on the horizontal axis.

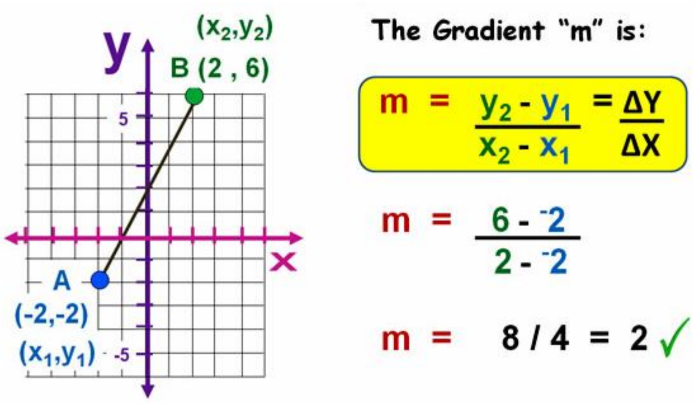
Correct language:
Gradient, rate of change, variable.

Step-by-step process:

The gradient is calculated by working out the change in y values divided by the change in x values. This can be written mathematically as:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta Y}{\Delta X}$$

Example:



Statistics

3.1 Categorical/Qualitative data

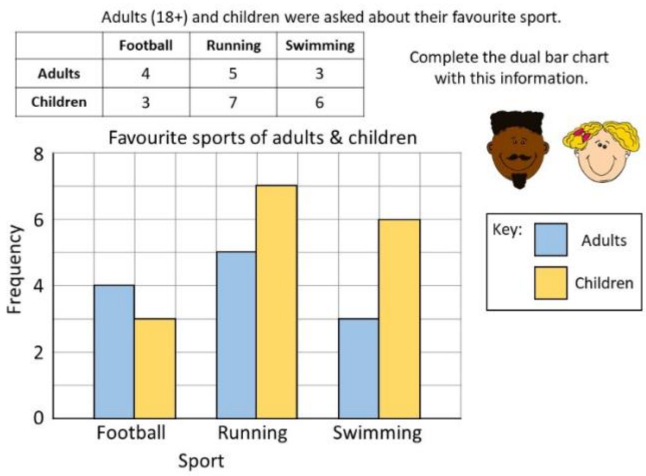
Categorical data are data that can be sorted into categories (e.g. different ‘eye colours’ or ‘food groups’) but cannot be ordered (since they are ‘labels’ that have no particular order). Categorical data are qualitative data.

Bar Charts & Line Graphs

A bar/line chart is a display for presenting data, in which bars/lines of equal width represent the set of values. Each value is proportional to the length of the bar. The bars/lines may be vertical or horizontal. Both axes must be labelled and there must be an appropriate scale used with equal intervals. A key must be used on the comparative bar charts – compound/composite/stacked and dual/grouped/clustered. Bar charts and line charts can be used for categorical or discrete data.

<u>Correct language:</u>
Compound/composite/stacked bar chart = bars are stacked on top of each other representing separate factors within each category
Dual/Grouped/Clustered bar chart = the bars are drawn in pars/groups and the two bars in each group represent separate factors within each category.

<u>Step-by-step process:</u>

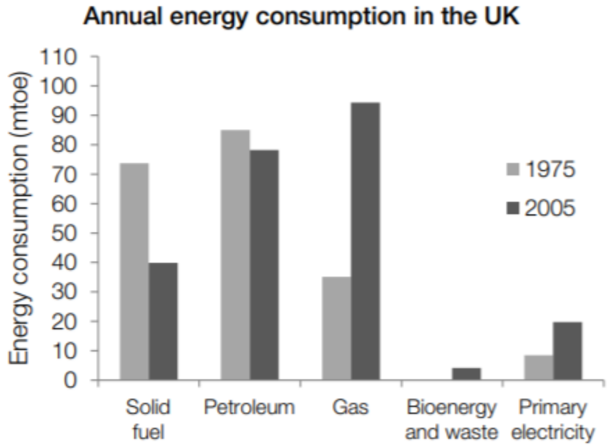

Adults (18+) and children were asked about their favourite sport.

	Football	Running	Swimming
Adults	4	5	3
Children	3	7	6

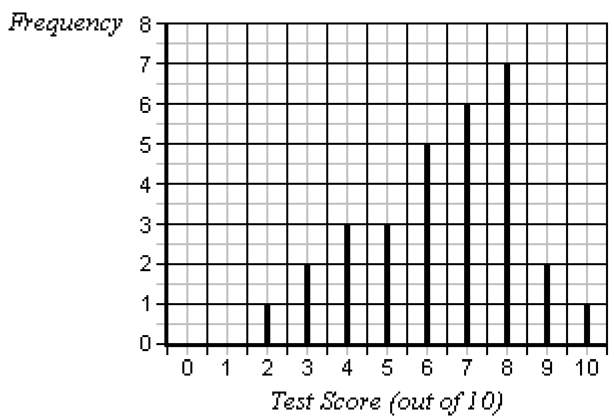
- Equal size bars with equal gaps between
- Both axes labelled
- Appropriate scale used with equal gaps

<u>Example:</u>

Dual bar chart:



Vertical line chart:



3.2 Numerical data

Numerical data refers to the data that is in the form of numbers, and is often referred to as quantitative data. Continuous and Discrete data are both types of numerical data.

Groups, Frequency Diagrams and Frequency Polygons

Correct language:

Numerical, quantitative, continuous, discrete, frequency, groups/intervals/classes

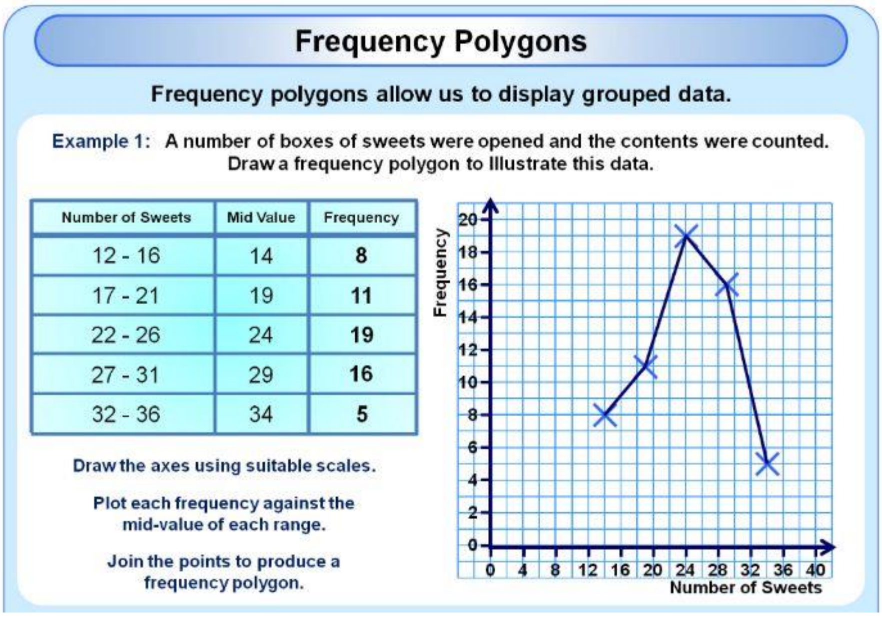
Step-by-step process:

Below are the different types of graphs that students may come across:

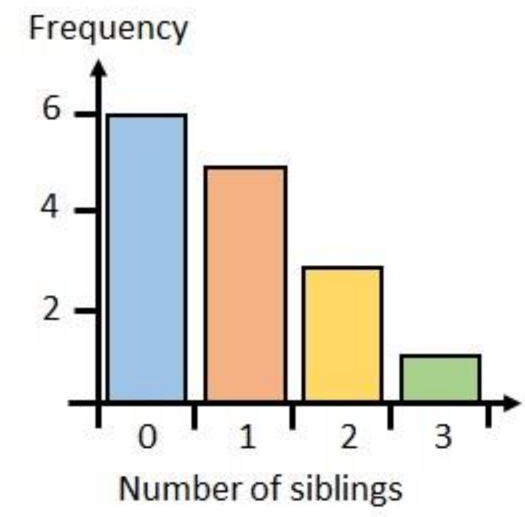
Example:

Grouped frequency table:

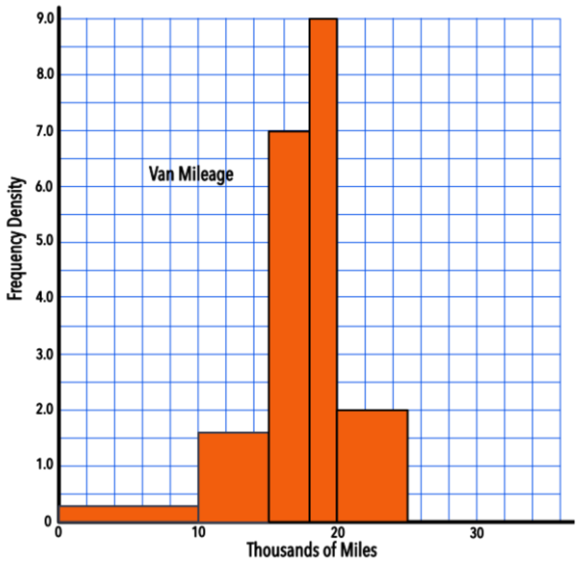
Weight, w (kg)	Frequency, f
$2 < w \leq 3$	22
$3 < w \leq 3.5$	14
$3.5 < w \leq 4$	39
$4 < w \leq 4.5$	29
$4.5 < w \leq 6$	13



Bar Chart:



Histogram:



3.3Scatter Diagrams and Lines of best fit

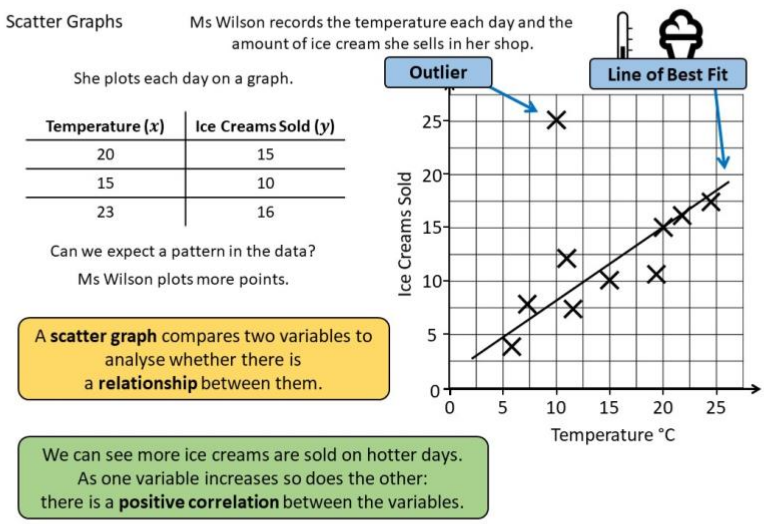
Scatter graphs/diagrams are used to present the relationship between two variables for a set of data (bivariate data), eg. Height and arm span. The data can be described as having a positive, negative or no correlation.
A line of best fit can be used to best represent the data and then used. In maths the line of best fit tends to be linear (a straight line); in Science the line of best fit may also be curved. The line/curve must have a similar number of data points either side of it.

Correct language:
Bivariate, variables, relationship, correlation, line of best fit.

Step-by-step process:

To draw a scatter graph use the steps outlined below:
Step 1: Plot each pair of data against each other, mark a cross where the piece of data lies
Step 2: Draw a line of best that best represents the data – it must have a similar number of data points either side of it. A ruler must be used if a straight line is drawn.
Step 3: To estimate another value from the scatter graph the line of best fit must be used. Use a dotted line to estimate from the line of best fit.

Example:



3.4 Pie Charts

A pie chart is a type of graph in which a circle is divided into sectors that each represent a proportion of the whole.

Correct language:
Sectors, proportion, angles, frequency.

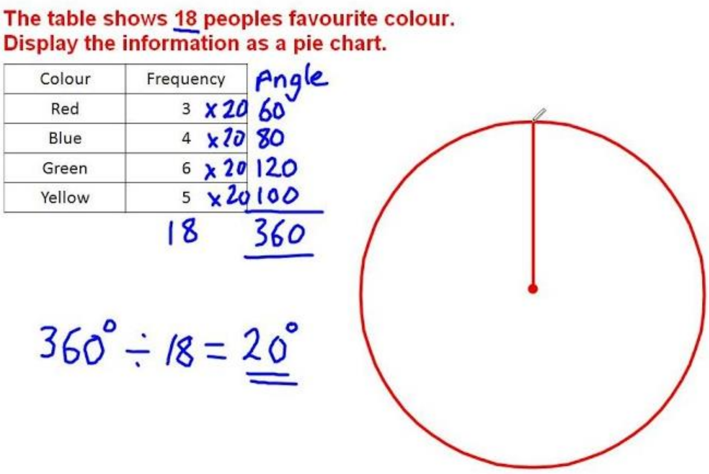
Step-by-step process:

Drawing a pie chart:
Step 1: Find the total frequency by finding the sum of the frequencies.
Step 2: Number of degrees per person/part = 360 ÷ total frequency
Step 3: Multiply each frequency by the degrees per person/part to find the number of degrees for that section
Step 4: Draw the pie chart using a protractor and a ruler

Interpreting a pie chart:
To find frequencies from a pie chart use the method outlined below.

Frequency of section = $\frac{degrees\ in\ section}{360} \times total\ frequency$

Example:



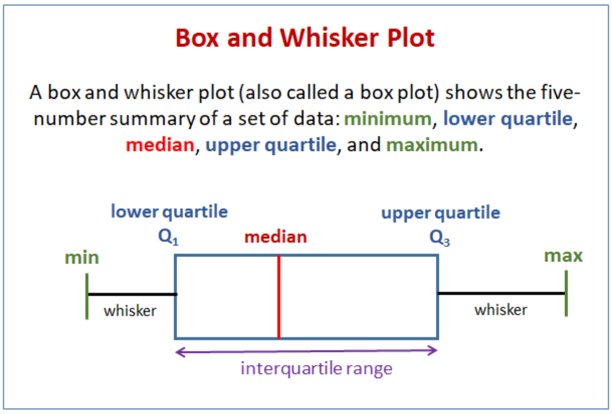
A protractor is then used to draw each angle on to the pie chart.

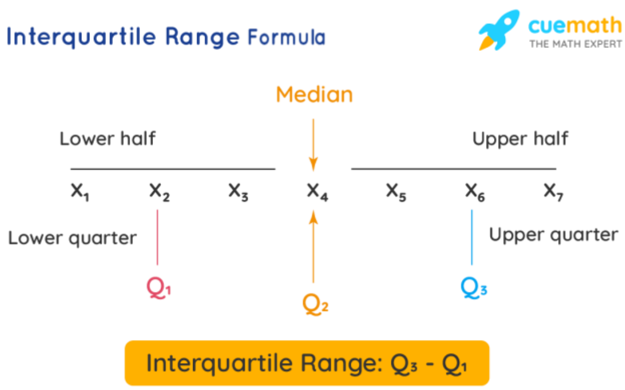
3.5 Box Plots & Interquartile Range

A box plot is a simple way of representing statistical data on a plot in which a rectangle is drawn to represent the second and third quartiles, usually with a vertical line inside to indicate the median value. The lower and upper quartiles are shown as horizontal lines either side of the rectangle.
The interquartile range (IQR) measures the spread of the middle half of your data. It is the range for the middle 50% of your sample. Use the IQR to assess the variability where most of your values lie. Larger values indicate that the central portion of your data spread out further. Conversely, smaller values show that the middle values cluster more tightly.

<u>Correct language:</u>
Quartiles, lower quartile, upper quartile, median, sample, spread, average.

<u>Step-by-step process:</u>

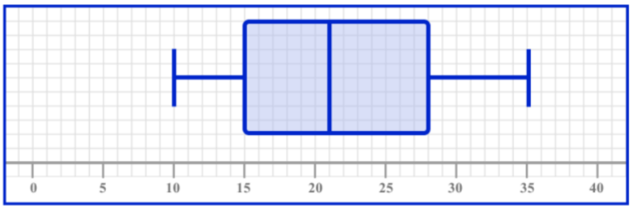




<u>Example:</u>

Draw a box plot using the following information.

Lowest Value	10
Lower Quartile	15
Median	21
Upper Quartile	28
Highest Value	35



Lower half | Upper half

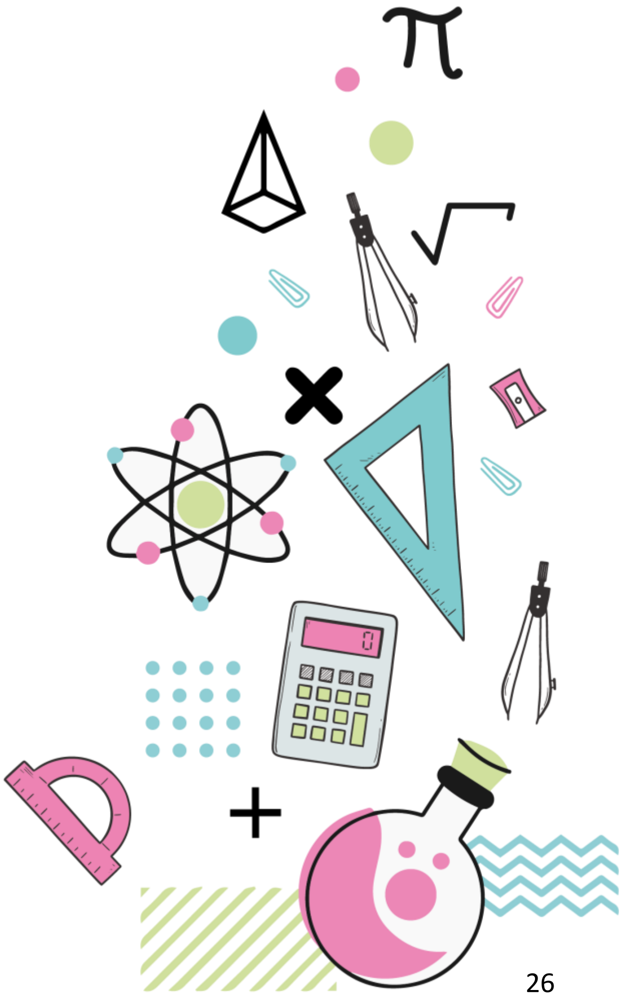
45, 47, 52, 52, 53, 55, 56, 58, 62, 80

Median

$$\frac{53 + 55}{2} = 54$$

$Q_1 = 52$ $Q_3 = 58$

Interquartile Range = $Q_3 - Q_1 = 58 - 52 = 6$



3.6 Primary, Secondary, and Tertiary data

Raw data is data collected directly from experiments or surveys, before being processed. Primary data is data collected directly by the user – the raw data once it has been processed. Secondary data is obtained indirectly from sources such as books, articles or web pages. Tertiary data is based on a collection of primary and secondary data eg. in a textbook.

Correct language:

Data, primary, raw, secondary, tertiary, advantages, disadvantages.

Step-by-step process:

Determine which type of data has been used or is best by the definitions above. Possibly may need to weigh up the advantages and disadvantages of each of the main types of data outlined below:

Advantages of primary data

- Knowledge of how the data was collected
- Knowledge of how reliable the data is
- Flexibility

Disadvantages of primary data

- Time
- Cost
- Potential limit of sample size

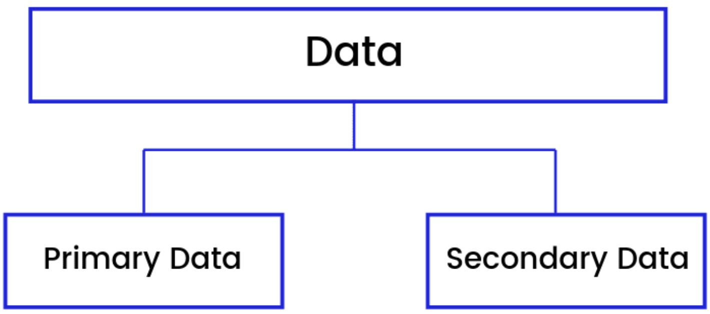
Advantages of secondary data

- Ease of obtaining
- Inexpensive

Disadvantages of secondary data

- May be unreliable
- Unknown collection methods

Example:



Examples:
1. Interviewing people within your organization
2. Observing a moderated discussion amongst people you choose to collect data

Examples:
1. Google Analytics report on your website's traffic
2. Information you collect from census and electoral statistics

3.7 Sampling

A sample is defined as a smaller set of data that a researcher chooses or selects from a larger population by using a pre-defined selection method.

Correct language:
Sample, population, data, random, systematic, stratified.

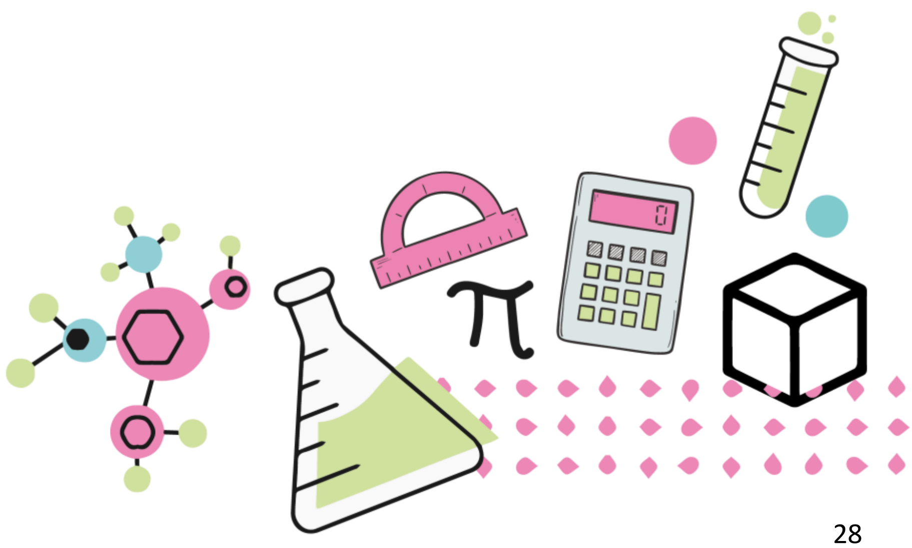
Step-by-step process:
There are three main sampling types that are described below.
Random sampling is the sampling technique in which each sample has an equal probability of being chosen.
Stratified sampling is a method of sampling that involves the division of a population into smaller groups/categories. The proportion of things from each category is the same in the sample and the population.
Systematic sampling can be used when the population is large. You choose a random starting point and take a sample at regular intervals – e.g. every 10th member of the population.

Example:

Adam wants to choose 20 of the 89 members of his choir to fill in a questionnaire.
Explain how he could select a random sample.

Everyone should have the same chance of being chosen,
so he needs to start with a list of everyone in the population...

1. First, he should make a list of all the people in the choir and assign everyone a number.
2. Then he could use a calculator or computer to generate 20 random numbers.
3. Finally, he needs to match these numbers to the people on the list to create the sample.



Geometry

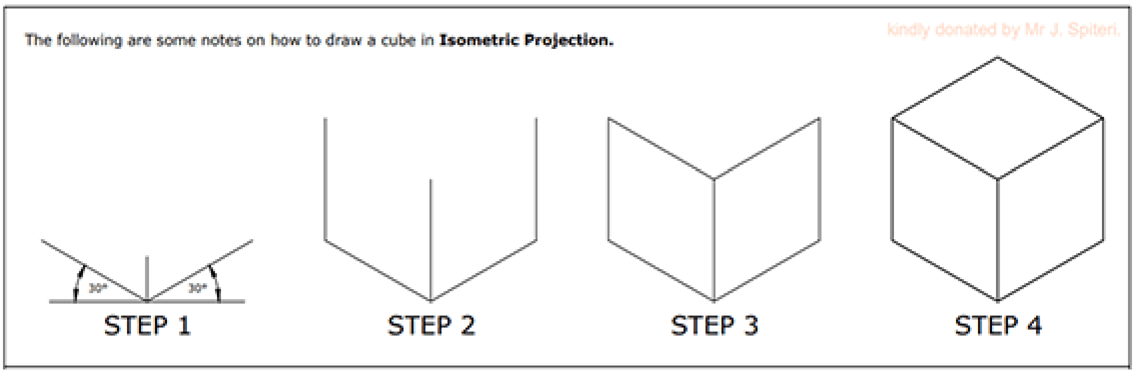
4.1 Isometric Drawing

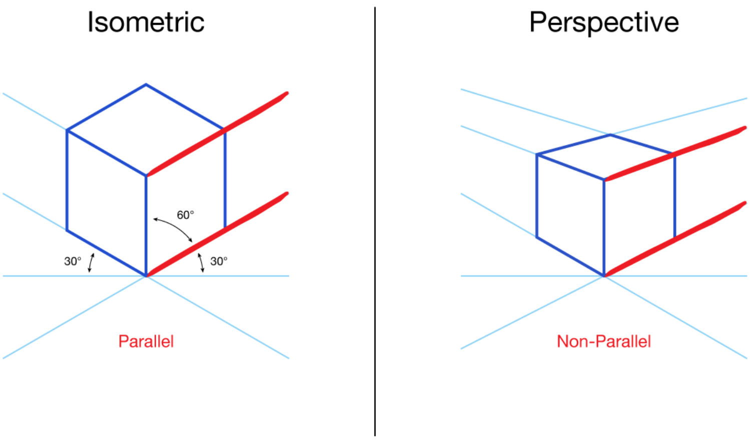
Isometric drawing is a form of 3D drawing, which is set out using 30-degree angles.

Correct language:

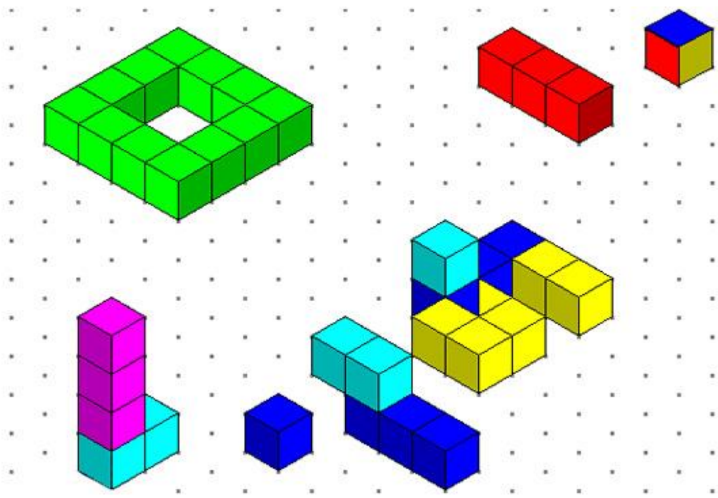
Isometric, 3-Dimensional, parallel.

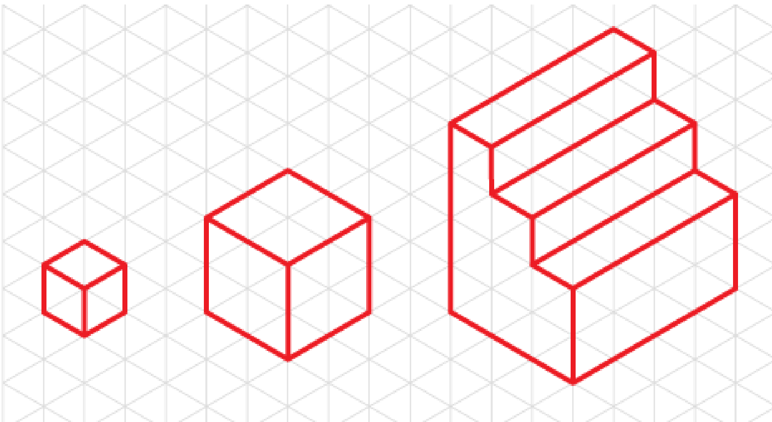
Step-by-step process:





Example:





4.2 Mensuration (Perimeter, Area and Volume)

Perimeter is the distance around a two-dimensional shape, area can be defined as the space occupied by a flat shape or the surface of an object, and volume is the amount of space that is contained within an object or solid shape.

Correct language:

Perimeter, area, volume, length, width, base, height, depth, surface.

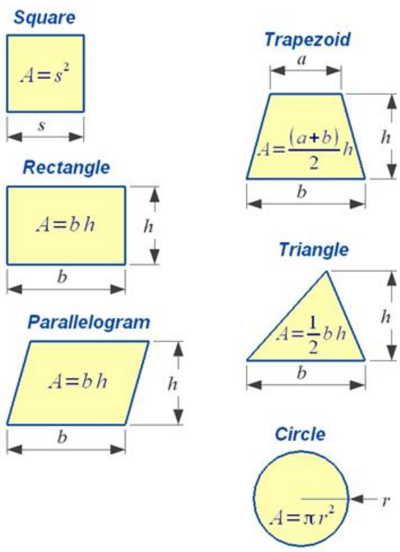
Step-by-step process:

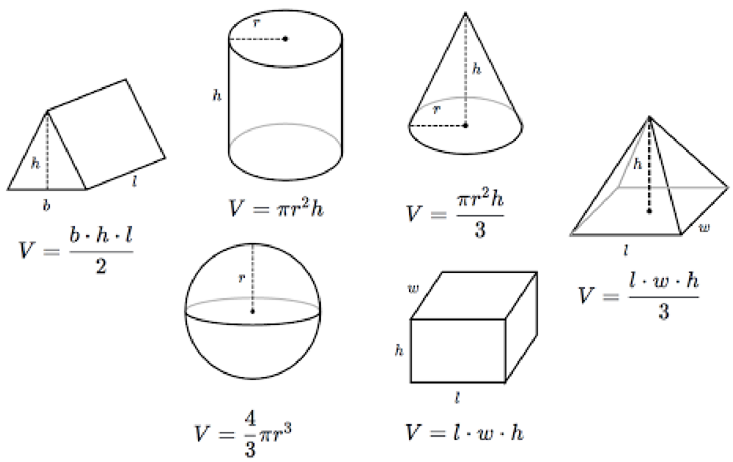
Perimeter, area and volume are to be calculated by using the set formulae outlined below.

Perimeter = the sum of the lengths of all of the edges of a shape.

Area formulae:

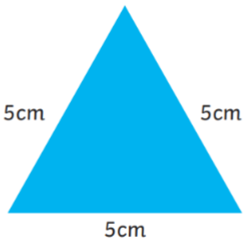
Volume formulae:





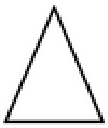
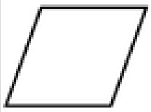
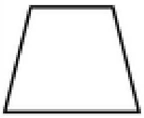
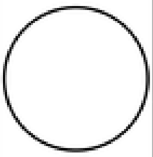
Example:

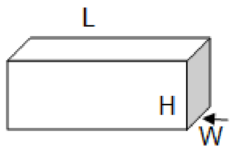
Perimeter:



5 + 5 + 5 = 15cm
Perimeter = 15cm

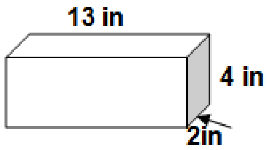
Area:

	Name	Formula	Example
	Rectangle	Area = Base x Height	Base = 4cm Height = 5cm Area = $20cm^2$
	Triangle	Area = $\frac{\text{Base} \times \text{Height}}{2}$	Base = 4cm Height = 5cm Area = $10cm^2$
	Parallelogram	Area = Base x Height	Base = 4m Height = 5m Area = $20m^2$
	Trapezium	Area = $\frac{(A + B) \times \text{Height}}{2}$ A and B are the parallel sides	A = 3m, B = 7m Height = 6m Area = $30m^2$
	Circle	Area: $\pi \times r^2$ R = radius	R = 4mm Area = $50.3mm^2$
		Circumference = $\pi \times d$ D= diameter	R = 4mm Circumference = 12.6mm



Volume = L • W •H
Volume = Length • Width• Height

Example:



Volume = L • W •H
Volume = 13 cm • 2 cm • 4 cm
Volume = 104 cm^3

Volume: